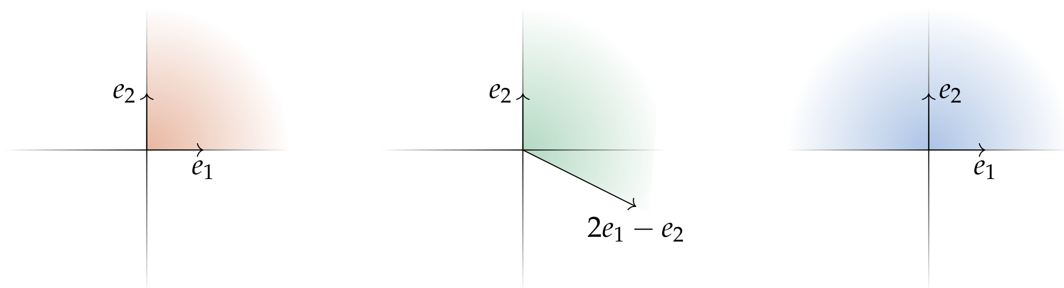
Exercise 1. *For each of the following cones:*

(1) Write the generators and compute the dimension.
(2) Compute the dual cone.
(3) List all the faces of σ, expressed as $\sigma \cap u^{\perp}$ for some u in the dual ambient vector space. Do the same for $\check{\sigma}$.
(4) Verify the orientation-reversing bijection $\tau \leftrightarrow \tau^*$, and the dimension formula $\dim(\tau) + \dim(\tau^*) = n$, where n is the dimension of the ambient space.



Bonus: Try the following case in $\mathbb{R}^3$, where the cone is generated by e_1, $-e_1$, $e_2 + e_3$, and $-e_2 + e_3$.

Exercise 2. *Compute the generators of the monoid $\sigma \cap \mathbb{Z}^2$, where σ is the green cone above.*

Exercise 3. *Let $\sigma \subset \mathbb{R}^n$ be a cone, and let $M \subset \mathbb{R}^n$ be a lattice. Gordon's lemma states that if σ is a rational cone, then the monoid $\sigma \cap M$ is finitely generated. Can you provide an example of a non-rational cone for which $\sigma \cap M$ is not finitely generated?*

Exercise 4. *Let $\sigma = \{0\}$ be the trivial cone in $\mathbb{Z}^n \subset \mathbb{R}^n$. Compute the associated monoid S_σ, the associated algebra R_σ, and determine the corresponding variety.*