ETHZ – Toric Geometry – Spring 2025 **Organiser: Alessandro Giacchetto**
Exercise sheet 2

Exercise 1. *Consider the cone σ in $\mathbb{R}^2$ generated by e_1 and $3e_1 - 2e_2$. Describe $\check{\sigma}$, find generators of $S_\sigma = \check{\sigma} \cap \mathbb{Z}^2$, compute the toric ideal of the affine variety X_σ, and describe the torus in X_σ.*

Exercise 2. *Consider the cone σ in $\mathbb{R}^3$ generated by e_1, e_2 and $e_1 + e_2 + 2e_3$. Describe $\check{\sigma}$, find generators of $S_\sigma = \check{\sigma} \cap \mathbb{Z}^3$, and compute the toric ideal of the affine variety X_σ, and describe the torus in X_σ.*

Exercise 3. *Let $\sigma \subset \mathbb{R}^n$ be a cone. Prove that the following are equivalent.*

- *σ is strongly convex.*
- *$\{0\}$ is a face of σ.*
- *σ contains no positive-dimensional subspace of $\mathbb{R}^n$.*
- *$\sigma \cap (-\sigma) = \{0\}$*
- *$\dim \check{\sigma} = n$.*